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GENDER DISCRIMINATION, EDUCATION AND ECONOMIC GROWTH IN A GENERALIZED UZAWA-LUCAS TWO-SECTOR MODEL

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This paper is mainly concerned with relationships between economic growth and gender discrimination in labor markets and education. Although discrimination in different fields has well been addresses and modelled in the economic literature, there are only a few growth models with endogenous wealth and human capital accumulation, gender time distribution between work, leisure and education under gender (positive or negative) discrimination. The production and economic structures, human capital accumulation are based on the Uzawa-Lucas model, while the utility function and gender division of labor, leisure time and study time are based on the model by Zhang. The model takes account of learning by education in modeling human capital accumulation. We simulate the model to demonstrate the existence of equilibrium points and motion of the national economy. We also conduct a comparative dynamic analysis in regard to changes in discrimination in the education sector, women's propensity to stay at home, women's propensity to receive education, women's knowledge utilization efficiency, and the propensity to save.

Keywords: Economic Growth; Gender; Gender-Differentiated Time Distribution; Learning By Education; Propensity To Receive Education.

JEL Classification: D2, L2, M1.

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1. Introduction

Gender discrimination is still widespread and persistent in modern economies. Flabbi (2010: 745) observes: "Even if wages and earnings for women and men in the United States have experienced a significant convergence in the 1970s and 1980s, their ratio has remained roughly constant at 75% since the mid-1990s... The United States is not an exception among OECD countries: they rank more or less average, with Northern European countries traditionally showing the lowest differentials and Japan the highest. These differentials persist after conditioning on observable productivity characteristics, and the consensus in the literature is that a significant portion of the conditional differential remains 'unexplained.'" Since the seminal study on gender discrimination was published by Becker (1957), economists have studied gender discrimination extensively both in formal theories and empirical research. In his important work Becker concludes that as competition in product becomes more intensive, costly discrimination will become weaker. Black and Brainerd (2004: 541) observe, "The recent narrowing of the gender earnings gap in an era of increased competition through international trade and deregulation might seem to offer support for this theory. Since 1960, the time trend for the female: male wage ratio has closely tracked that for imports as a share of GDP, with both series remaining fairly constant between 1960 and 1980, then increasing dramatically through the early 1990s".

It is often argued that taste for gender discrimination is not sustainable as firms' indulgence in discrimination is too expensive. Gap reduction between female and male earnings may occur due to many other reasons, such as education, policy against discrimination, and work experiences (e.g., Goldin, 1990; O'Neil and Polachek, 1993; Blau and Kahn, 1997; Blau, 1998). Black and Brainerd (2004) study the impact of economic globalization on gender discrimination in manufacturing industries. The research focuses on how changes in the competitive environment will lead to changes in the gender wage differential. They find that trade may benefit women by reducing the ability of firms to discriminate. In a recent study on pay gap between men and women in Turkey, Akhmedjonov (2012: 32) concludes that "estimated wage gap between men and women in Turkey is almost entirely explained by labor market discrimination toward women." In a recent study on gender discrimination in U.S. public relations, Dozier et al. (2013) try to identify the determinants of female and male income differentials in public relations. They find five factors that explain income disparity in public relations. They include years of professional experience, manager role enactment, participation in management decision-making, income-suppressing career interruptions, and career specialization. Dozier et al. (2013: 13) conclude: "However, even with all these mediating variables accounted for, the average income was $84,368 for men and $76,063 for women, a difference of $8,305. With no other mediating variables tested that can account for this residual income difference, we argue that $8,305 annually (or $332,200 over a 40-year career) is the

concrete cost of gender discrimination in public relations." They also emphasize: "Our research continues to confirm what the past three decades of empirical studies have shown us: women earn less simply because of their gender." Dozier et al. (2013: 13). Chen et al. (2013) study the link between globalization and gender inequality in China. They find that domestic firms employ less female workers than foreign and exporting firms. Within the same region and industry firms with foreign participation and export orientation reduce the gender wage gap. They observe gender wage discrimination only among private nonexporting firms. They conclude that globalization in China tends to encourage female employment and reduce gender discrimination. Patterson and Walcutt (2014) study why gender discrimination has been continued in South Korea workplaces, even though South Korea has carried out gender policy reforms and improved female education over decades. They identify factors such as a lack of legal enforcement, an acceptance of the status quo by women, a weak punishment status as well as culture-related mind-set. There are also many other studies on differential issues related to gender discrimination and labor market conditions (e.g., Oaxaca, 1994; Serini et al. 1998; Antecol and Kuhn, 2000; Jolliffe, 2002; Hutton, 2005; Agnes, et al., 2013; Bartolucci, 2013; Zuzana and Lenka, 2013; Kuhn and Shen, 2013). These studies confirm the existence of gender discrimination in developed economies as well as in developing economies.

Different theoretical ideas and models about discrimination in labor market are proposed. Becker (1957) holds that discrimination occurs because of a taste for discrimination on the part of employers, workers or consumers. This idea is incorporated in many models of labor market (e.g., Bowlus, 1997; Heckman, 1998; Altonji and Blank, 1999; Bowlus and Eckstein, 2002). Flabbi (2010) recently proposes a search model with employers' taste discrimination. The model makes it possible to separately identify gender discrimination and unobserved productivity differences. Prejudice is measured by the disutility that a proportion of employers receive when employing female workers. Arrow (1972) uses the concept of statistical discrimination to describe the situations that employers are uncertain about individual qualities of workers. Francois (1998) builds a model of gender discrimination in competitive labor market. The model takes account of the interaction between men and women within the household. In reviewing approaches to wage discrimination from economic perspectives, Stiglitz (1973: 287) raises the following question: "Under what circumstances is it possible for groups with identical economic characteristics to receive different wages in a market equilibrium? If people of the same productivity receive different wages, then there are profits to be made by hiring the low-wage individual. If all firms are profit maximizers, then all will demand the services of the low-wage individuals, bidding their wages up until the wage differential is eliminated. Why does this not occur?" Stiglitz also presents different models which are built on alternative hypotheses. As emphasized by Stiglitz, different models may be appropriate for explaining different discriminations. There are, for instance, racial discrimination, gender discrimination, and religious or ethnic discrimination. Different from Becker's and Arrow's hypotheses, Vlassis and Drydakis (2012) study discrimination in industries where equally skilled workers who are members of firm-specific monopoly unions can be grouped by reservation wages. In their

approach employers are unbiased against any particular group and any individual and there is no taste discrimination. They confirm the possible existence of equilibrium of discriminatory wage contracts across groups of employers under either an oligopoly or a perfectly competitive product market. Although there are different approaches to discrimination in general, and gender discrimination in particular, one of the central problems in the economics of gender discrimination has not been properly examined within a general equilibrium framework with endogenous wealth and education. The purpose of this study is to introduce gender discrimination into a general equilibrium model with endogenous wealth and human capital.

The observed wage gaps between men and women are affected by many factors such as national wealth, education and human capital, as well as gender discrimination. It is obviously necessary to take account of education and discrimination within a single framework. A dramatic change in modern history is the entry of women into the labor force. Bar and Leukhina (2011) observe that married females more than doubled their workforce participation in the last half a century. Many empirical studies have been carried out about labor market and economic development with gender (Viscusi, 1980; Blau and Kahn, 1992, 2000; Nachum, 1996; Croson and Gneezy, 2009; Biagetti and Sergio, 2009). Eckstein and Lifshitz (2011) find that improved education accounts for about 33 percent of the increase in female employment, and the rise in wages and narrowing of the gender wage gap account for another 20 percent, while about the 40 percent remains unexplained by observed household characteristics. In many economies firms have become more willing to hire women due to factors such as the rise of service sectors, anti-discrimination policy measures, the increase of higher education among women, and the development of new technologies. On the other hand, women tend to work more in labor markets as consequences of lower fertility rates, the availability of new household technologies, and the emergence of flexible-time work. Many formal economic theories are proposed to deal with the changes in female labor participation take place and examine the factors which are significant determinants of the dynamics (e.g., Becker, 1965, 1985; Chiappori, 1992; Hadfield, 1999; Gomme et al. 2001; Campbell and Ludvigson, 2001; Gutierrez, 2003; Tassel, 2004; Fernández, 2007; Nosaka, 2007; Trede and Heimann, 2011). In particular, time distribution between home and non-home economic and leisure activities have caused much attention (e.g., Greenwood and Hercowitz, 1991; Benhabib and Perli, 1994; Ladrón-de-Guevara et al. 1997; Turnovsky, 1999; Rupert et al. 2001; Vendrik, 2003). Although there are many studies on gender differences in education and growth, as pointed out by Bandiera and Natraj (2013), these studies are of limited use for revealing relationships between education and growth as they often do not identify the causal link from gender differences to economic growth.

In contemporary economics human capital is a key determinant of economic growth (e.g., Hanushek and Kimko, 2000; Barro, 2001; Krueger and Lindahl, 2001; Castelló-Climent and Hidalgo-Cabrillana, 2012). It is necessary to analyze the dynamic interdependence between economic growth and human capital in order to properly understand contemporary economies. It is necessary to develop a theory of gender discrimination within a general analytical

framework with endogenous wealth and human capital. Inspired by the richness of empirical studies and influenced by different formal economic models, this study develops an integrated analytical framework to study endogenous labor supply and gender division of labor with gender discrimination. We follow the growth model with gender-differentiated human capital and family wealth accumulation proposed by Zhang (2014). Zhang's model is a synthesis of neoclassical growth theory and the Uzawa-Lucas two-sector model. In Zhang's approach physical capital accumulation is based on the neoclassical growth theory. Human capital accumulation is modeled according to the approach by Uzawa (1965) and Lucas (1988). The Uzawa-Lucas model has been further developed in different directions (for instance, Jones et al. 1993; Stokey and Rebelo, 1995; Mino, 1996, 2001; Alonso-Carrera and Freire-Seren, 2004; De Hek, 2005). Although there are many models of economic growth with endogenous human capital and knowledge (see also, Schultz, 1981; Romer, 1986; Grossman and Helpman, 1991; Aghion and Howitt, 1998), a few formal growth models with human capital are developed with gender differences. Zhang (2014) extends the Uzawa-Lucas two-sector model to include gender difference. The model by Zhang differs from almost of the theoretical economic models with gender in that it integrates endogenous human capital, physical capital, and elastic labor supply of man and woman within a comprehensive framework. We introduce gender discrimination into Zhang's model. We introduce some new dynamic relations between variables which are not addressed in the economic growth literature with gender division of labor and gender discrimination. The paper is organized as follows. Section 2 introduces the basic model with gender discrimination, wealth accumulation and human capital accumulation. Section 3 simulates the model. Section 4 carries out a comparative dynamic analysis with regard to some parameters. Section 5 concludes the study.

2. The Basic Model

The basic framework of the model is the same as the model by Zhang (2014) which is an extension of the Uzawa-Lucas model. Most of this section is essentially the same as section 2 in Zhang (2014) except parts related to gender discrimination. The economy consists of one production and one education sector. The production sector is the same as in the Solow model (e.g., Solow, 1956; Burmeister and Dobell, 1970; Azariadis, 1993; Barro and Sala-i-Martin; 1995). Saving is undertaken and assets are owned only by households. Exchanges take place in perfectly competitive markets. Factor markets work well and factors are fully utilized at every moment. We select the commodity to serve as numeraire, with all the other prices being measured relative to its price. The family supplies labor services, recreation, spiritual experiences, as well as conventional goods of the do-it-yourself variety. The population of each gender is homogeneous. We assume that each family consists of husband and wife. As all the families are identical, the family structure is invariant over time. We follow the same spirit as described by Albanesi and Olivetti (2009: 82): "Since the purpose of this paper is to study the joint determination of gender differentials in labor market outcomes and in the household division of labor, we abstract from modelling marriage decisions …".

We use subscripts $q=1$ and $q=2$ to stand for man and woman respectively. The population is constant. We use $\bar{N}$ to stand for the population of each gender. Let $T_1(t)$ and $T_2(t)$ stand for the work time of husband and wife of a representative household and $N(t)$ for the flow of labor services used at time t for production. We have $N(t)$

$$N(t) = \left[H_1^{\theta_1}(t)T_1(t) + H_2^{\theta_2}(t)T_2(t)\right]\bar{N},$$

where $H_q(t)$ is the level of human capital of gender q and θ_q is gender q's human capital utilization efficiency parameter. We call $H_q^{\theta_q}(t)$ gender q's level of effective human capital.

The labor force is distributed between the two sectors. The total stock of physical capital $K(t)$ is fully employed by the two sectors. We use $N_e(t)$ and $K_e(t)$ to stand for the labor force and capital stocks employed by the education sector, and $N_i(t)$ and $K_i(t)$ for the labor force and capital stocks employed by the production sector. As labor and capital are assumed fully employed, we have

$$K_i(t) + K_e(t) = K(t), \quad N_i(t) + N_e(t) = N(t). \tag{1}$$

The production sector

The production function is

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \quad A_i, \alpha_i, \beta_i > 0, \ \alpha_i + \beta_i = 1, \tag{2}$$

where A_i is the total productivity of the production sector, and α_i and β_i are respectively the constant output elasticities of capital and qualified labor input. Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. We denote $w_q(t)$ the wage rate per unit of work time of gender q in the fair labor market. In the fair labor market a worker is paid according to the worker's marginal value of labor.

As we consider the labor market with gender discrimination, we have to specify how discrimination is conducted (in addition to the literature cited in the Introduction, see also, Heyman, et al. 2013; Jonathan and Kerwin, 2013; Lanning, 2014). For simplicity of analysis we assume that a fraction ϕ of women's fair share of the gender's labor is redistributed to men in the same industry. We call ϕ the discrimination rate against woman in the labor market. It should be noted that it is difficult to know the value of the discrimination rate. Dozier et al. (2013: 13) point out: "Gender discrimination cannot be measured by directly asking

respondents in a survey if they systematically discriminate against women with regard to salaries. Since such conduct is illegal, that question would yield only normative responses. Thus, we are left with the somewhat unsatisfactory methodology of testing any variables that might mediate the relationship between gender and income. We treat the residual variance as a quantified estimate of gender discrimination." Let $N_{iq}(t)$ stand for the qualified labor force of gender q by the industrial sector. The total cost of the female labor force due to discrimination against woman is $\phi\, w_2(t)N_{i2}(t)$. We have the industry's profit as follows

$$F_i(t) - (r(t) + \delta_k)K_i(t) - w_1(t)N_{i1}(t) - (1 - \phi)w_2(t)N_{i2}(t)$$

where δ_k is the depreciation rate of physical capital. The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)}, \quad w_1(t) = \frac{\beta_i H_1^{\theta_1}(t) F_i(t)}{N_i(t)}, \quad w_2(t) = \frac{\beta_i H_2^{\theta_2}(t) F_i(t)}{(1-\phi)N_i(t)}. \tag{3}$$

The education sector

There are different types of discrimination in economic systems. As Stiglitz (1973: 288) points out, "Previous analyses have erred in not exploring the implications of discriminatory preferences in a general equilibrium framework with more than one sector." In our approach education is treated as a service sector. There are empirical researches on education discrimination in different forms (e.g., Ouazad and Page, 2013). The education sector has the same discrimination rate against female teachers as the industrial sector. Except gender discrimination in the teacher market, we assume that there is also gender discrimination in education fee. We use $p(t)$ to stand for the fair education fee per unit of time. The total education service is measured by the total education time received by the population. The production function of the education sector is assumed to be a function of $K_e(t)$ and $N_e(t)$ as follows

$$F_e(t) = A_e K_e^{\alpha_e}(t) N_e^{\beta_e}(t), \ \alpha_e, \beta_e > 0, \ \alpha_e + \beta_e = 1, \tag{4}$$

where A_e, α_e and β_e are positive parameters. As for the industrial sector, the marginal conditions for the education sector are

$$r(t) + \delta_k = \frac{\alpha_e \hat{p}(t) F_e(t)}{K_e(t)}, \quad w_1(t) = \frac{\beta_e \hat{p}(t) H_q^{\theta_q}(t) F_e(t)}{N_e(t)}, \quad w_2(t) = \frac{\beta_e \hat{p}(t) H_2^{\theta_2}(t) F_e(t)}{(1-\phi)N_e(t)}, \tag{5}$$

where $\hat{p}(t)$ is the average fee per unit of education time that the education sector charges. We will define this variable late on.

Behavior of consumers

We use an alternative approach to modelling consumer behaviour proposed by Zhang (1993). The households make choice of consumption levels of services and commodities, education time and leisure time as well as on amount of saving under the specified discrimination. We use $\bar{k}(t)$ to stand for wealth per household, i.e., $\bar{k}(t) = K(t)/\bar{N}$. The per household current income $y(t)$ from the interest and wage payments as follows

$$y(t) = r(t)\bar{k}(t) + w_1(t)T_1(t) + (1 - \phi)w_2(t)T_2(t).$$

The sum of money that consumers are using for consuming, saving, or transferring are not necessarily equal to the current income because consumers can sell wealth to pay, for instance, current consumption if the temporary income is not sufficient for purchasing goods and services. The total value of wealth that a representative household can sell to purchase goods and to save is equal to $\bar{k}(t)$. We assume that selling and buying wealth can be conducted instantaneously without any transaction cost. This is evidently a strict consumption as it may take time to draw savings from bank or to sell one's properties. The per capita disposable income of the household is defined as the sum of the current income and the wealth available for purchasing consumption goods and saving

$$\hat{y}(t) = y(t) + \bar{k}(t).$$

Following Zhang (2014) at each point in time, the household would distribute the total available budget between saving, $s(t)$, consuming good, $c(t)$, and receiving education, $\tilde{T}_1(t)$, and $\tilde{T}_2(t)$. We assume that the representative female student is charged an extra amount of education fee $\phi_e\, p(t)$. We call ϕ_e the discrimination rate against woman in education. The budget constraint is given by

$$c(t) + s(t) + p(t)\tilde{T}_1(t) + \bar{\phi}_e\, p(t)\tilde{T}_2(t) = \hat{y}(t), \qquad (6)$$

where $\bar{\phi}_e \equiv 1 + \phi_e$. This equation means that consumption and savings exhaust the consumers' disposable income. Let $\bar{T}_q(t)$ stand for the leisure time of gender q at time t. A person of gender q is faced with the time constraint

$$T_q(t) + \tilde{T}_q(t) + \bar{T}_q(t) = T_0,$$

where T_0 is the total available time for work, education and leisure. Substituting this function into (6) yields

$$c(t) + s(t) + \underset{1}{p}(t)\tilde{T}_1(t) + \underset{2}{p}(t)\tilde{T}_2(t) + w_1(t)\bar{T}_1(t) + \bar{\phi}\, w_2(t)\bar{T}_2(t) = \bar{y}(t), \qquad (7)$$

where $\bar{\phi} \equiv 1 - \phi$ and $\hat{y}(t) \equiv y(t) + k(t)$, and

$$\bar{p}_1(t) \equiv p(t) + w_1(t), \quad \bar{p}_2(t) \equiv \bar{\phi}_e\, p(t) + \bar{\phi}\, w_2(t),$$
$$\bar{y}(t) \equiv \left(r(t) + 1\right)\bar{k}(t) + w_1(t)T_0 + \bar{\phi} w_2(t)T_0.$$

The left-hand side of (7) is the sum of the cost of consumption, saving, and opportunity costs of leisure and education times. At each point in time, the household decides six variables: the level of consumption, the level of saving, the leisure times, and the education times.

This study considers differences in human capital and preference between men and women. Stotsky (2006) empirically finds gender differences and effects of these differences on economic variables. In this study we consider these differences by a household utility function. We assume that the utility level $U(t)$ is dependent on the education times, the leisure times, the consumption level of commodity, and the saving as follows

$$U(t) = u(t)\bar{T}_1^{\sigma_{01}}(t)\bar{T}_2^{\sigma_{02}}(t)\tilde{T}_1^{\eta_{01}}(t)\tilde{T}_2^{\eta_{02}}(t)c^{\xi_0}(t)s^{\lambda_0}(t), \ \sigma_{0q}, \eta_{0q}, \ \xi_0, \lambda_0 > 0,$$

where u is a time-dependent variable, σ_{0q} and η_{0q} are called respectively gender q's propensities to use leisure time and to receive education, and ξ_0 and λ_0 respectively the family's propensities to consume good and to hold wealth.

It should be noted that in a study of gender discimination by Francois (1998), each gender has the following utility function

$$U(c) + V(e + H)$$

in which U is consumption, e is effort at work, and H is effort in the household. In this approach an outsider may also be employed to do housing job. Our study uses an aggregate household utility function to describe behavior of both genders. Some feasures of the approach to behavior by Francois can be applied to refine our model. We may also make our model more relevant by taking account of some factors in modelling gender choice of education by Echevarria and Merlov (1999). They explain gender differences in education as the equilibrium outcome of a gender-differentiated overlapping generations model. In their model men and women of each generation make decision jointly on consumption, number of children, and expenditures in education of their children. There are empirical evidences on gender

discrimination against children education within households (Kingdon, 2005; Chaudhuri and Roy, 2006; Kumar, 2013; Saha, 2013). We simplify the household decision by omitting issues related to children.

Maximizing U subject to budget constraint (7) yields

$$w_q(t)\bar{T}_q(t) = \sigma_q \bar{y}(t),\ \bar{p}_q(t)\tilde{T}_q(t) = \eta_q \bar{y}(t),\ c(t) = \xi \bar{y}(t),\ s(t) = \lambda \bar{y}(t), \tag{8}$$

where

$$\rho \equiv \frac{1}{\sigma_{01} + \sigma_{02} + \eta_{01} + \eta_{02} + \xi_0 + \lambda_0},\ \xi \equiv \rho \xi_0,\ \lambda \equiv \rho \lambda_0,\ \sigma_1 \equiv \rho \sigma_{01},\ \eta_1 \equiv \rho \eta_{01},$$
$$\sigma_2 \equiv \frac{\rho \sigma_{02}}{\bar{\phi}},\ \eta_2 \equiv \rho \eta_{02}.$$

The expenditure on each item is equal to the potential disposable income by the item's propensity to consume.

There are historical studies of discrimination (Folbre, 2009). As observed by Antecol (2000), there is considerable variation in the gender gap in labor participation rates (LFPR) across countries. "[T]he gender gap in LFPR, which is the male LFPR minus the female LFPR, ranges from 89.4 percentage points for Afghanistan, 50.6 percentage points for Greece, to 2.2 percentage points for Sweden. Perhaps surprisingly, there is little work among economists that attempts to explain cross-country variation in female labor force participation rates." From (9), we see that the ratio of education time between man and woman is positively related to the ratio of man's and woman's propensity to receive education, negatively related to the ratios of man's and woman's economic costs of education. As education cost of a gender is relate to the wage rate, the tax rate on wage, and the price of education, the ratio is affected by many factors. From (8), we have

$$\frac{\bar{T}_1(t)}{\bar{T}_2(t)} = \frac{(1-\bar{\phi})\sigma_{01}}{\sigma_{02}} \frac{H_2^{\theta_2}(t)}{H_1^{\theta_1}(t)},\quad \frac{\tilde{T}_1(t)}{\tilde{T}_2(t)} = \frac{\eta_1}{\eta_2} \frac{\bar{\phi}_e p(t) + \bar{\phi} w_2(t)}{p(t) + w_1(t)}, \tag{9}$$

where we also use $w_1(t)/w_2(t) = H_1^{\theta_1}(t)/H_2^{\theta_2}(t)$. We see that the ratio of time at home between husband and wife is positively related to the ratio of man's and woman's propensity to stay at home. A higher discrimination rate in labor education tends to reduces the ratio as strengthened discrimination against women in labor market makes women have stronger economic incentive to stay at home. Our analysis coincides with the conclusion of the empirical study by Albanesi and Olivetti (2009: 81), "gender differences in earnings and the fraction of performance pay are positively related to the gender differences in home hours."

Wealth accumulation

We now find dynamics of capital accumulation. According to the definition of $s(t)$, the change in the household's wealth is given by

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t), \tag{10}$$

where implies that change in wealth is saving minus dissaving.

Balance of demand and supply

The total demand for education $\tilde{T}(t)$ is the sum and male and female demand for education. That is

$$\tilde{T}(t) = \tilde{T}_1(t)\bar{N} + \tilde{T}_2(t)\bar{N}.$$

For the education sector, the demand for and supply of education balance at any point in time

$$\tilde{T}(t) = F_e(t). \tag{11}$$

As output of the production sector is equal to the sum of the level of consumption, the depreciation of capital stock and the net savings, we have

$$C(t) + S(t) - K(t) + \delta_k K(t) = F_i(t), \tag{12}$$

where $C(t)$ is the total consumption, $S(t) - K(t) + \delta_k K(t)$ is the sum of the net saving and depreciation and

$$C(t) = c(t)\bar{N}, \quad S(t) = s(t)\bar{N}. \tag{13}$$

The average price of education

The average price is the price that the education sector receives for per unit time of education it provides. As women and men pay different prices because of gender discrimination, the average price is different from the price that each gender pays. We now define the variable of average price $\hat{p}(t)$ as follows

$$\hat{p}(t) = \frac{\tilde{T}_1(t)\bar{N}}{\tilde{T}(t)} p(t) + \frac{\tilde{T}_2(t)\bar{N}}{\tilde{T}(t)} (1 + \phi_e) p(t). \tag{14}$$

If there is no gender discrimination, the average price equals the fair price in the education market. By (14) we also have

$$\hat{p}(t) = p(t) + \frac{\tilde{T}_2(t)\bar{N}}{\tilde{T}(t)}\phi_e\, p(t).$$

The average price equals the fair price plus the extra charge due to discrimination. We see that the revenue of the education sector is $\hat{p}(t)F_e(t)$.

Accumulation of human capital

In this study, we follow the Uzawa-Lucas model in modeling human capital accumulation. We assume that human capital accumulation is through education. Let $\tilde{T}_1(t)$ and $\tilde{T}_2(t)$ stand for the education time of husband and wife of a representative household. We propose the following human capital accumulation equation (Zhang, 1993, 2014)

$$\dot{H}_q(t) = \Gamma_{qe}(t) - \delta_{qh} H_q(t), \quad q = 1, 2, \tag{15}$$

Where

$$\Gamma_{qe}(t) \equiv \frac{\upsilon_{qe}\left(F_e(t)/2\bar{N}\right)^{a_{qe}}\left(H_q^{\theta_q}(t)\tilde{T}_q(t)\right)^{b_{qe}}}{H_q^{\pi_{qe}}(t)},$$

where δ_{qh} (>0) is the depreciation rate of human capital, υ_{qe}, a_{qe}, and b_{qe} are non-negative parameters. The term $\Gamma_{qe}(t)$ are contributions to gender q's human capital, respectively, through education and learning by doing. Human capital tends to increase with an increase in the level of education service, $F_{qe}/2\bar{N}$, and in the (qualified) study time, $H_q^{\theta_q}T_{qe}$. The term $H_q^{\pi_{qe}}$ indicates that as the level of human capital of the population increases, it may be more difficult (in the case of π_{qe} being large) or easier (in the case of π_{qe} being small) to accumulate more human capital via formal education. We will simulate the model when returns to scale are not strong.

We have thus built the dynamic model. The model is general in the sense that the well-known models such as the Solow one-sector growth model, the Uzawa two-sector growth model, the Uzawa-Lucas two-sector growth model, and Zhang's two-sector model with gender division of labor are special case of the model developed in this section. We now examine properties of the dynamic model.

3. The Dynamics and Its Properties

This section examines the dynamics of the model. First, we introduce a new variable

$$z(t) \equiv \frac{r(t)+\delta_k}{w_1(t)}.$$

We show that the dynamics can be expressed by the three-dimensional differential equations system with $z(t)$, $H_1(t)$, and $H_2(t)$ as the variables.

Lemma

The dynamics of the economic system is given by the three-dimensional differential equations

$$\begin{aligned} \dot{H}_q(t) &= \Omega_q\left(z(t), H_1(t), H_2(t)\right), \quad q = 1, 2, \\ \dot{z}(t) &= \Omega\left(z(t), H_1(t), H_2(t)\right), \end{aligned} \tag{16}$$

where Ω_q and Ω are functions of $z(t)$, $H_1(t)$ and $H_2(t)$ defined in the *Appendix*. Moreover, all the other variables are determined as functions of $z(t)$, $H_1(t)$ and $H_2(t)$ at any point in time by the following procedure: $\bar{k}(t)$ by (A23) $\rightarrow p(t)$ by (A10) $\rightarrow r(t)$ and $w_1(t)$ by (A4) $\rightarrow w_2(t)$ by (A2) $\rightarrow \hat{p}(t)$ by (A5) $\rightarrow T_1(t)$ and $T_2(t)$ by (A20) $\rightarrow N(t)$ by definition $\rightarrow K_e(t)$ by (A15) $\rightarrow K_i(t)$ by (A16) $\rightarrow N_e(t)$ and $N_i(t)$ by (A17) $\rightarrow K(t) = \bar{k}(t)N(t) \rightarrow F_j\left(K_j(t), N_j(t)\right) \rightarrow \bar{y}(t)$ by definition $\rightarrow$ $\bar{T}_q(t), \tilde{T}_q(t), c(t), s(t)$ by (8).

The system (16) contains three variables, $z(t)$, $H_1(t)$, and $H_2(t)$. The system is nonlinear. It is quite difficult to get analytical properties of the dynamic system. For simplicity, we simulate the model to illustrate behavior of the system. In the remainder of this study, we specify the depreciation rates by $\delta_k = 0.05$, $\delta_{jh} = 0.04$, and let $T_0 = 1$. The requirement $T_0 = 1$ will not affect our analysis. We specify the other parameters as follows
$N_0 = 100$, $\alpha_i = 0.35$, $\alpha_e = 0.3$, $A_i = 0.9$, $A_e = 0.9$, $\theta_1 = 0.6$, $\theta_2 = 0.57$, $\lambda_0 = 0.65$, $\xi_0 = 0.08$, $\eta_{01} = 0.015$, $\eta_{02} = 0.012$, $\sigma_{01} = 0.18$, $\sigma_{02} = 0.2$, $\phi = 0.03$, $\phi_e = 0.02$, $a_{qe} = 0.3$, $b_{qe} = 0.5$, $\pi_{qe} = 0.2$, $v_{1e} = 0.8$, $v_{2e} = 0.75$.

The propensity to save is 0.65 and the man's and woman's propensities to receive education are respectively 0.015 and 0.012 The propensity to consume goods is 0.08. The total

productivity factors of the two sectors are specified as $A_i = 1.2$ and $A_e = 0.9$. The conditions $\pi_{qe} = 0.2$ mean that the learning by education exhibits decreasing effects in human capital. The man's propensity to receive education is higher than the woman's. The man's propensity to stay at home is lower than the woman's. The condition $\theta_1 > \theta_2$ means that man applies human capital more effectively than woman. The condition $v_{1e} > v_{2e}$ means that the man's human capital accumulation is more effective than the woman's. We now specify the initial conditions to see how the gender-related variables change over time. To follow the motion of the system, we specify initial conditions

$$z(0) = 0.034, \quad H_1(0) = 1.65, \quad H_2(0) = 1.35.$$

Initially men's human capital level is higher than women's. We plot the simulation result in *Figure 2*. As the initial levels are fixed higher than their long-term equilibrium levels, men's and women's levels of the human capital fall over time. Men's and women's education times fall slightly over time. Men's and women's work hours rise slightly over time. Men's leisure time rises and women's leisure time falls. The total labor supply and total wealth, the capital and labor inputs and output levels of the two sectors fall. The rate of interest rises. The wage rates and the opportunity costs of education fall. The average price of education changes slightly over time.

From *Figure 1*, we observe that the variables tend to become stationary. The simulation confirms the existence of a locally stable equilibrium point. We calculate the equilibrium values of the variables as follows

$$N = 39.4, \quad K = 430.3, \quad H_1 = 1.41, \quad H_2 = 1.17, \quad N_i = 35.6, \quad N_e = 3.8,$$
$$K_i = 396.5, \quad K_e = 33.8, \quad F_i = 74.5, \quad F_e = 6.6, \quad r = 0.016, \quad \bar{p}_1 = 2.78, \quad \bar{p}_2 = 2.62,$$
$$\hat{p} = 1.11, \quad w_1 = 1.67, \quad w_2 = 1.53, \quad \bar{T}_1 = 0.71, \quad \bar{T}_2 = 0.89, \quad \tilde{T}_1 = 0.04, \quad \tilde{T}_2 = 0.03,$$
$$T_1 = 0.25, \quad T_2 = 0.08, \quad \bar{k} = 4.3, \quad c = 0.53.$$

It is straightforward to calculate the three eigenvalues

$$-0.205, \quad -0.035, \quad -0.021.$$

As the three eigenvalues are negative, the unique equilibrium is locally stable. Hence, the system always approaches its equilibrium if it is not far from the equilibrium. From *Figure 1*, we see that the system approaches its equilibrium point in the long term.

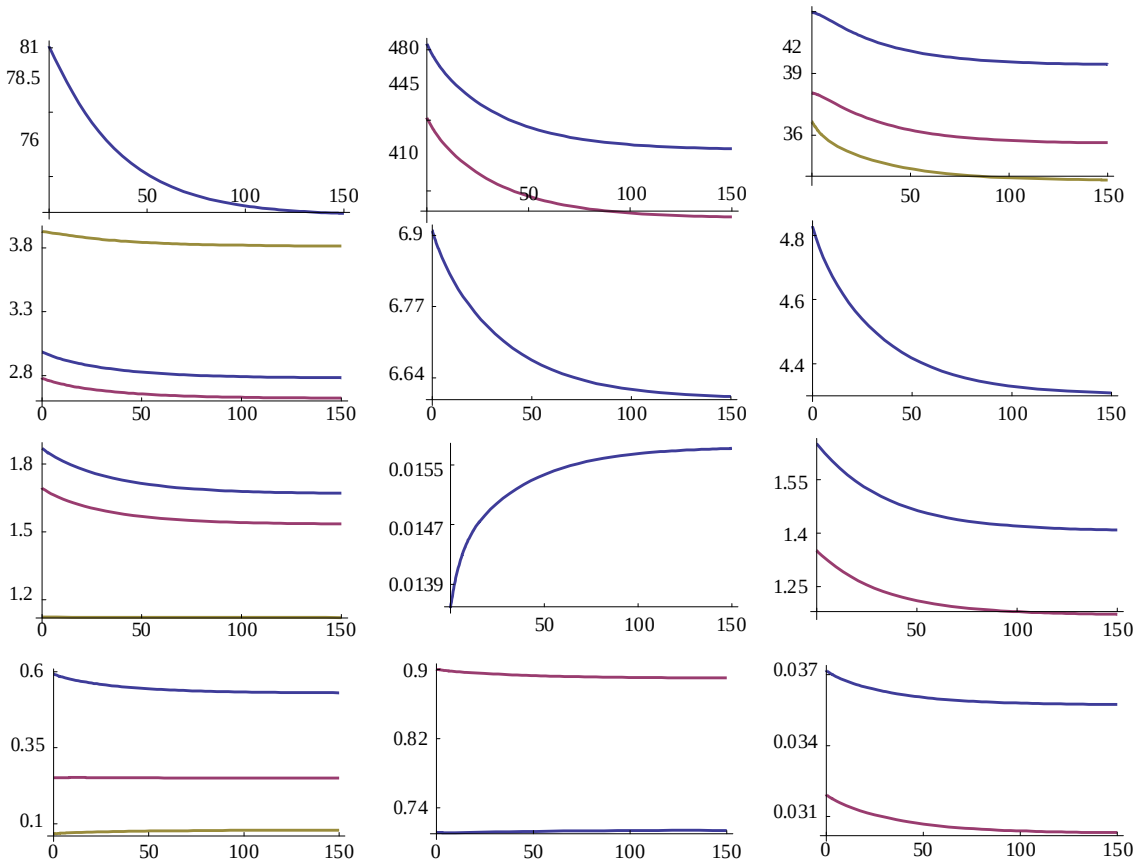

Figure 1. The Motion of the Economic System

4. Comparative Dynamics Analyses

We now examine impact of changes in a few parameters on dynamic processes of the system. We already simulated the motion of the national economy. As the lemma provides the computational procedure to calibrate the motion of all the variables, it is straightforward to examine effects of change in any parameter on transitory processes as well stationary states of all the variables. In order to examine how each variable is affected over time, we should follow the motion of the entire system as each variable is related to the others in the dynamic system. As the interactions are so complicated, it is verbally difficult to explicate how the variables interact over time in detail. We use a variable $\overline{\Delta}x(t)$ to stand for the change rate of the variable, $x(t)$, in percentage due to changes in the parameter value.

Increased discrimination against women in education

First, we examine the case that the discrimination rate against female students is increased as follows: $\phi_e : 0.02 \Rightarrow 0.06$. The simulation results are plotted in *Figure 2*.

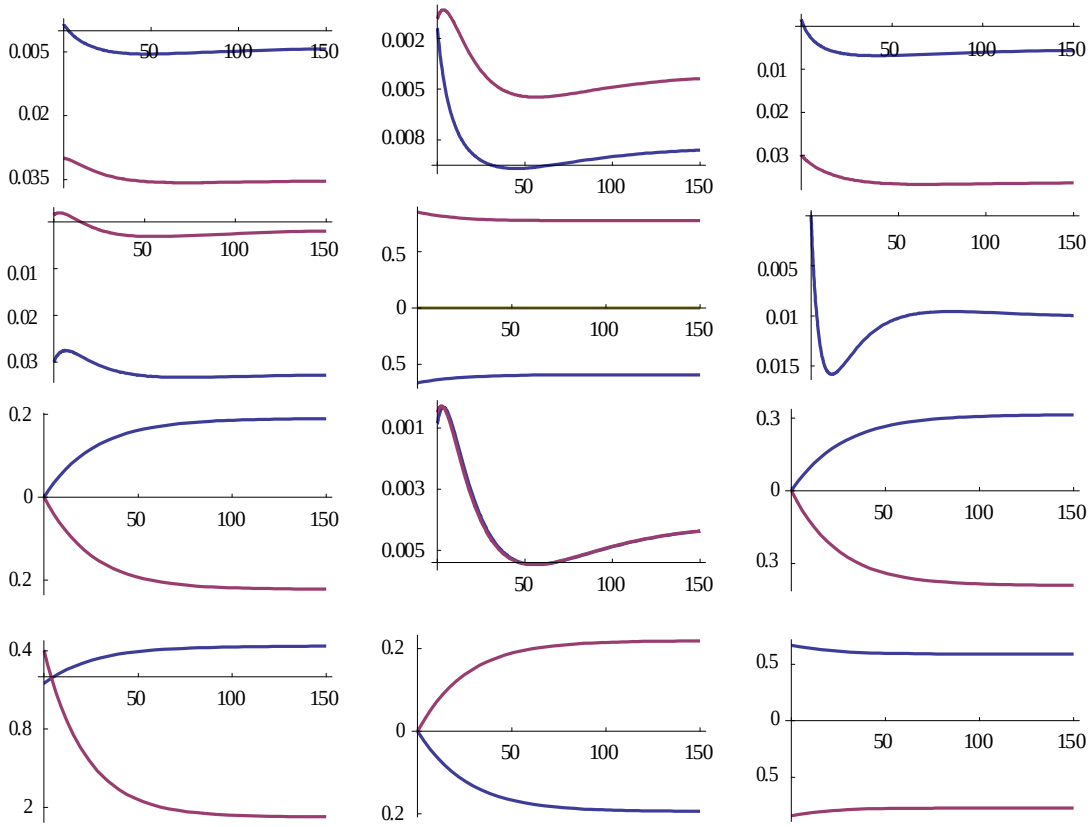

Figure 2. Increased Discrimination against Women in Education

A rise in the discrimination rate against women in education results in rises in women's opportunity cost of education. At the same time men's opportunity cost of education is reduced, even though the average price of education is slightly affected. Women reduce their study hours in association with the rising in women's opportunity cost, while men increase their study hours in association of falling in men's opportunity cost of education. Consequently men's human capital and wage rate are augmented, and women's human capital and wage rate are reduced. Women stay longer at home and work less, men stay less at home and work longer. The net consequence of the changes in human capital and time distribution between work, leisure and study time reduces the total labor supply. The

reduced amount is very slight. The labor inputs of the two sectors are reduced over time. The consumption level and wealth per household are reduced. This also implies falling of the total physical capital. The capital inputs and output levels of the two sectors are reduced. The rate of interest is reduced. We conclude that the discrimination against women has negative effects on the national wealth, national labor supply, output levels and consumption level per household. But the time that women stay at home is increased.

The impact of an augment in the total productivity of the education sector

We now consider the case that the education sector enhances its total factor productivity as follows: A_e : 0.9 $\Rightarrow$ 0.95. The changes in the motion are plotted in *Figure 3*.

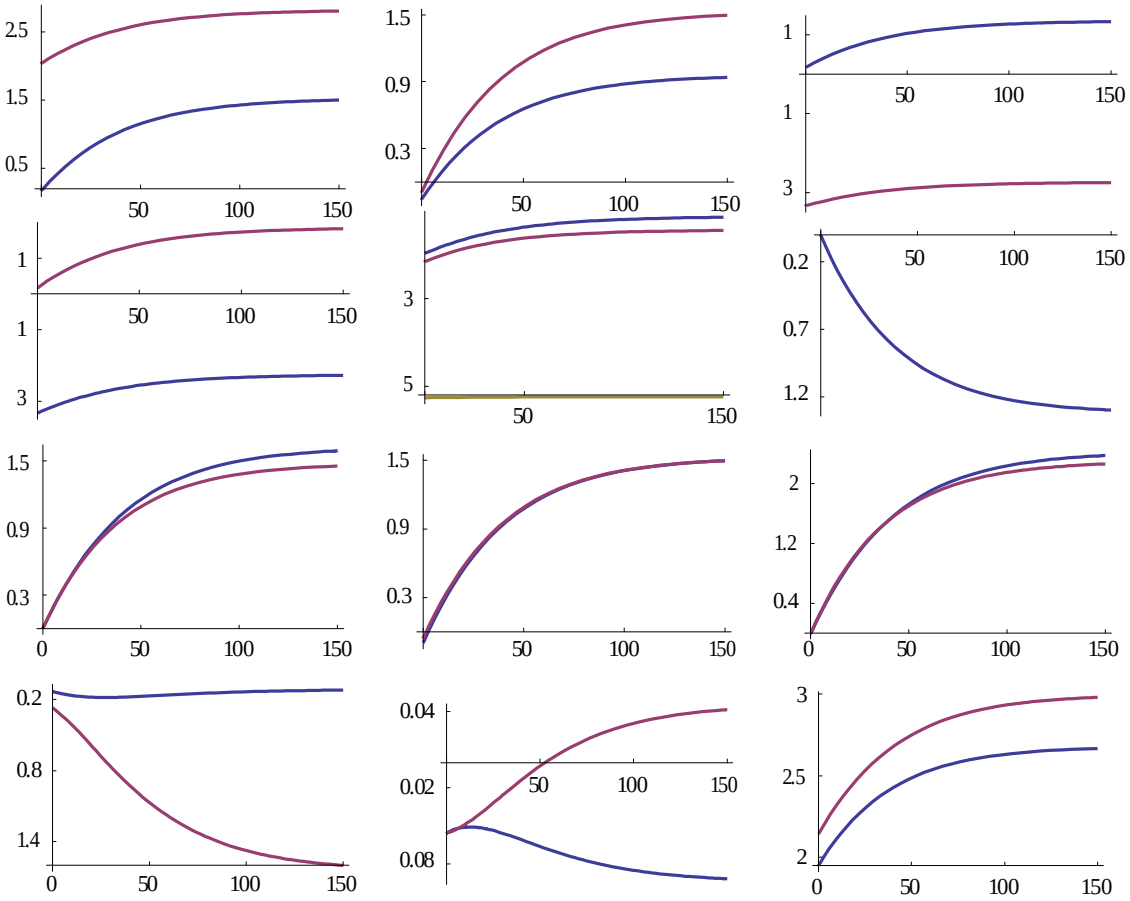

Figure 3. The Education Sector Enhancing the Total Factor Productivity

Source: Own preparation.

When the education sector enhances its productivity, the supply is increased. The average price and opportunity costs of woman and man are reduced. Women and men spend more hours on studying in association with the falling of opportunity costs of education. More study implies higher human capital levels and wage rates. Both women and men work less than before. Man's leisure time is reduced. Woman's leisure time is reduced initially but increased in the long term. Both consumption level and wealth are augmented. The total capital, total labor force and the two sectors' output levels are all increased. The education sector employs less capital and labor inputs, while the production sector employs more capital and labor inputs. The rise in the total physical capital reduces the rate of interest.

Gender discrimination increasing women's propensity to stay at home

Discrimination may be conducted in different ways. It is possible, for instance, that society does not have favorable attitude to women working outside the home. This kind of discrimination tends to enforce women to increase their propensity to stay at home. In their empirical research of patterns of women's work and determinants of the gender division of labor in rural Bangladesh, Bose et al. (2009) conclude that the gender division of labor is associated with both economic and socio-cultural factors. On the basis of studies of some Asian economies, Banerjee (1999) find that the state interventions and women's own changing perceptions have strong impact on gender relations in the household. We consider the following change in women's propensity to stay at home, σ_{02} : 0.2 $\Rightarrow$ 0.22. As women prefer more to staying at home than before, women's leisure time is increased and their work time and education time are reduced. Men work more than before and they spend less time on education and on leisure. Spending less time on study implies reduction of human capital of both men and women. The average education fee is slightly affected. In association with falling wage rates the opportunity costs of education are lowered both for men and women. The wealth and consumption levels are reduced. The national wealth, total labor force, capital and labor inputs and output levels of the two sectors are lowered. According to Stotsky (2006: 18), "the neoclassical approach examines the simultaneous interaction of economic development and the reduction of gender inequalities. It sees the process of economic development leading to the reduction of these inequalities and also inequalities hindering economic development." Our simulation shows that economic decline is associated with enlarged gender differences in wage rates, human capital and work time, even though we achieve the conclusion different from Stotsky's approach.

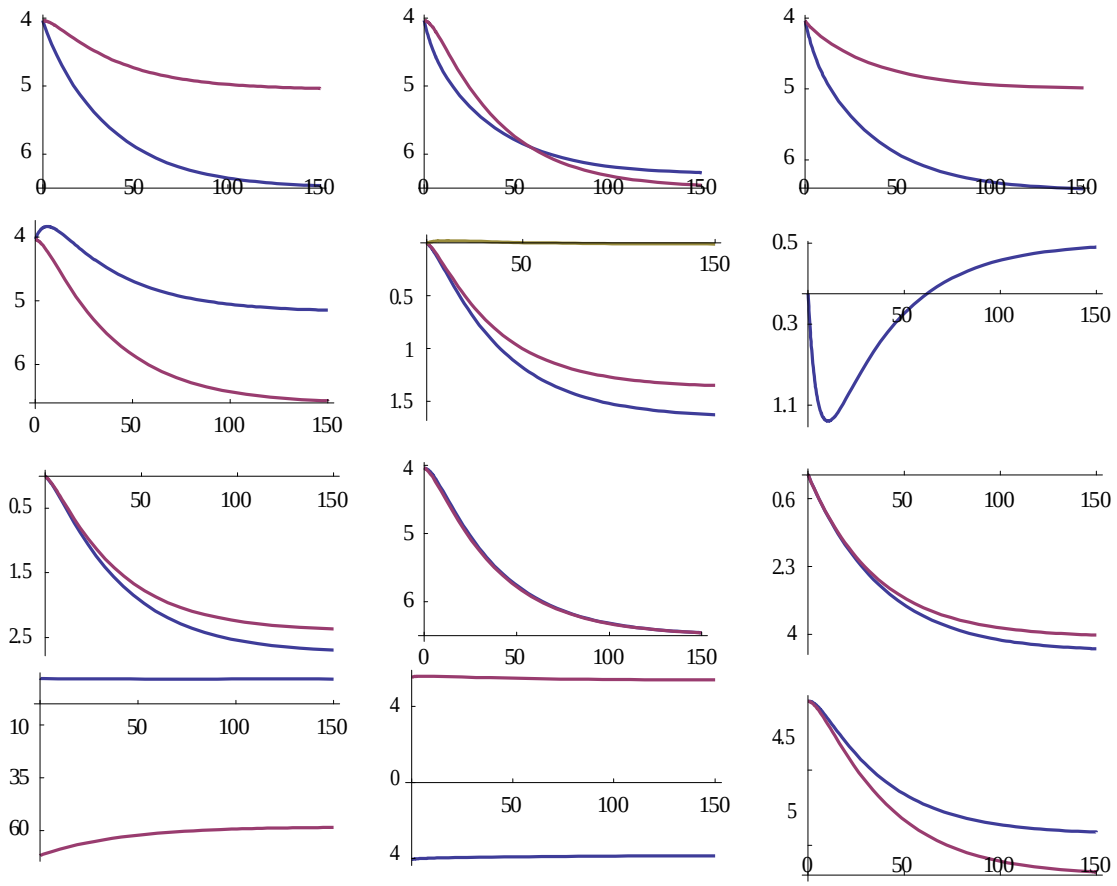

Figure 4. Discrimination Increasing Women's Propensity to Stay At Home

Women enhancing their propensity to receive education

It is argued by some researchers that gender inequalities due to disparities in human capital will wither away in association with economic development (e.g., Beneria and Feldman, 1992, Forsythe, et al. 2000). We are now interested in what happen when women have stronger incentive to get education, for instance, due to weakening in gender discrimination. We are concerned with the case that women enhance their propensity to receive education as follows: $\eta_{02} : 0.012 \Rightarrow 0.014$. We plot the simulation results in *Figure 5*. As women increase their propensity to receive education, they spend more time on study. Consequently women's human capital and wage rate are enhanced. Women spend less time on leisure and work longer. These changes increase the consumption and wealth levels. As wealth is increased, the wage rates of men and women are enhanced. The average cost of education is slightly affected. Hence,

women's and men's opportunity costs of education are increased. The national wealth, total labor force, capital and labor inputs and output levels of the two sectors are augmented. Our analysis also can provide other implications of an idea emphasized by Arrow (1973). Arrow argues that a risen interest in education may neither enhance human capital nor encourage economic growth. He reasons that students choose education also for the purpose of signaling (see also, Spence, 1973; Stiglitz, 1975). Lee (2007) argues that signaling may explain why American students study more in college than in high school while the opposite is true for East Asian students. The model of this study can explain the signaling consequence as well. If human capital accumulation is not efficient, then a rise in the propensity to receive education increases education time but not human capital and economic growth.

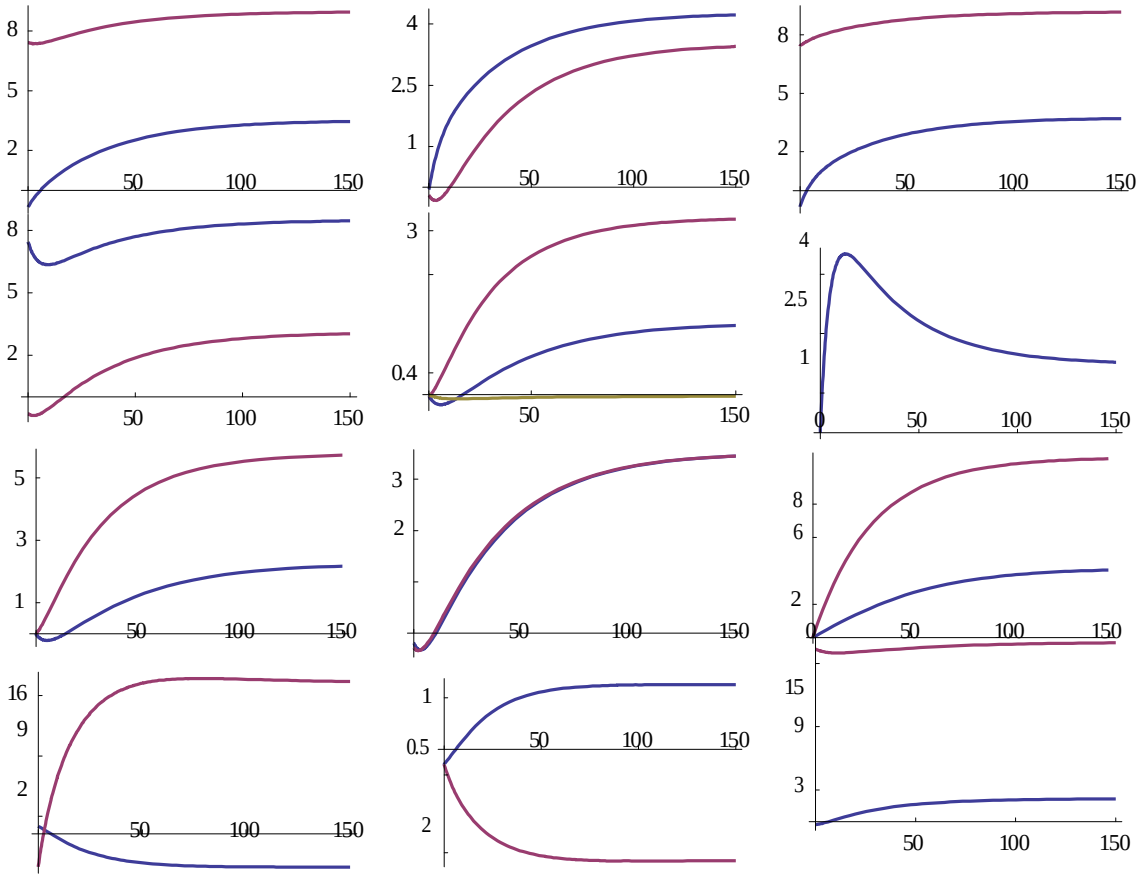

Figure 5 Women Enhancing Their Propensity to Receive Education

Woman's human capital utilization efficiency being enhanced

Boserup (1970) observes that there will be a curvilinear relationship between economic growth and the status of women (see also, Dolado, et al. 2001; Truong, 1997; Duflo, 2012). Boserup argued that initial stages of economic growth are characterized of a widening gap between men and women. Although productivity differences between women and men at low levels of economic development are not large, as economic conditions are improved, productivity differences tend to widen and a polarization and hierarchization of men's and women's work roles tend to ensure. Furthermore, the roles may be 'locked in' and possibly propagated by discrimination. Nevertheless, further economic growth will bring about a closing of the gap. The pace at which the gap is closed is dependent on many cultural, institutional, as well as economic factors (Becker, 1985; Fernández, 2007). Although there are few theoretical models related to gender distribution and economic growth within an integrated analytical framework, our model can be applied to address these issues in a consistent manner. Although gender difference in human capital utilization should be endogenous, our study treats the difference as exogenous. We now increase women's human capital utilization efficiency as follows: θ_1: 0.57⇒0.59 The results are plotted in *Figure 6*.

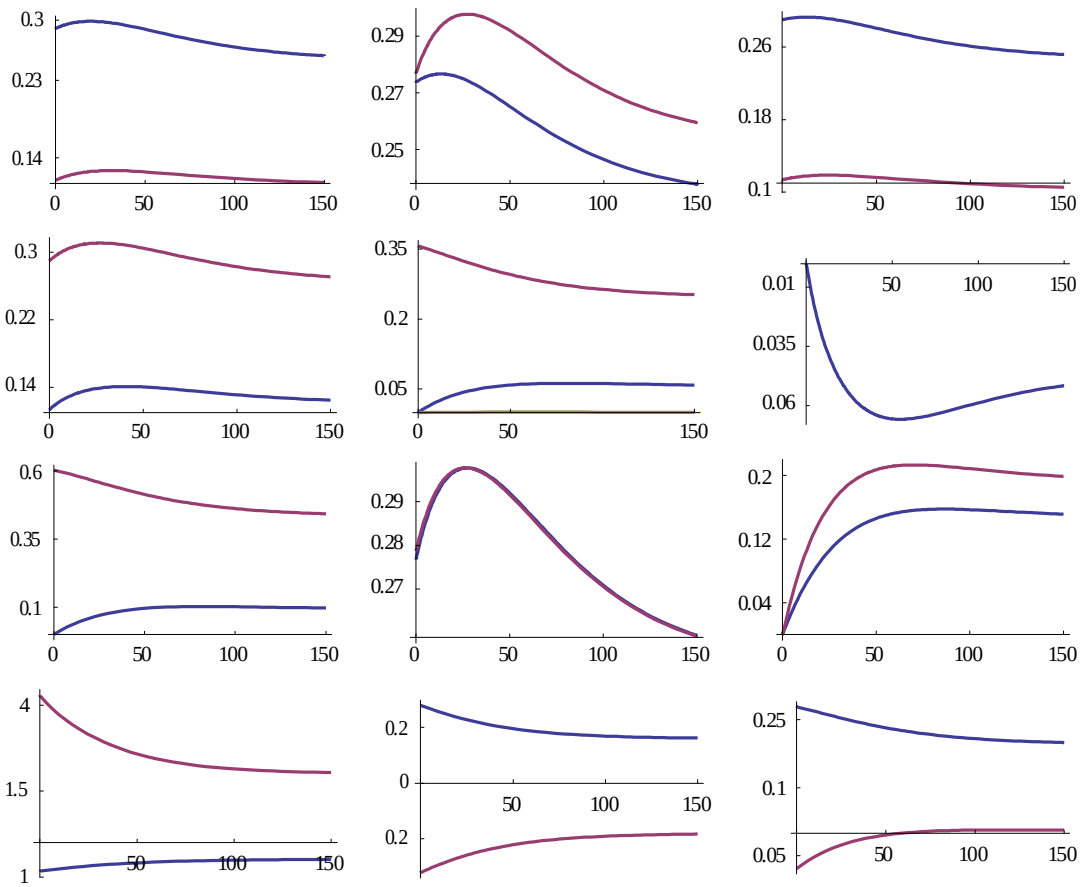

Figure 6. Woman's Human Capital Utilization Efficiency Being Enhanced

As the efficiency is improved, immediately women's wage rate is increased. This causes women's opportunity cost of education to be increased. Women's time on education is slightly affected. Women work more and have less leisure time as a consequence of rising in the opportunity cost of leisure time. As women use human capital more effectively, their human capital is improved. The rise in women's human capital also increases men's human capital as men spend more time on study. Men spend more time at home and work less. The average cost of education is slightly affected. Men's opportunity cost of education is increased. The national wealth, total labor force, capital and labor inputs and output levels of the two sectors are augmented.

The propensity to save being reduced

Education is another form of investment as current education brings about improved human capital which will enhance labor productivity. Investment ineducation also reduces current consumption or/and wealth accumulation. As pointed out by Chanda (2008) over the last three decades returns to higher education have increased while the household savings rate has fallen to almost zero in the US. Chanda constructs a dynamic model with the representative agent whose savings fall as an outcome of an exogenously driven increase in the return to education. We now examine the interaction between saving propensity and education within our analytical framework. We now reduce the propensity to save as follows: $\lambda_0 : 0.65 \Rightarrow 0.63$. The results are plotted in *Figure* 7. As the household puts less disposable income for wealth accumulation, the wealth per household falls. The consumption level rises initially but falls in the long term. The study times that men and women spend on education and the human capital levels rise initially but fall in the long term. The wage rates and average price of education are reduced. Consequently the opportunity costs of education both for men and women fall. The leisure hours of men and women are increased and work hours of men and women are reduced in association with falling wage rates. The national wealth, total labor force, capital input and output levels of the two sectors are reduced. The labor input of the production sector is reduced, while the labor input of the education sector is increased.

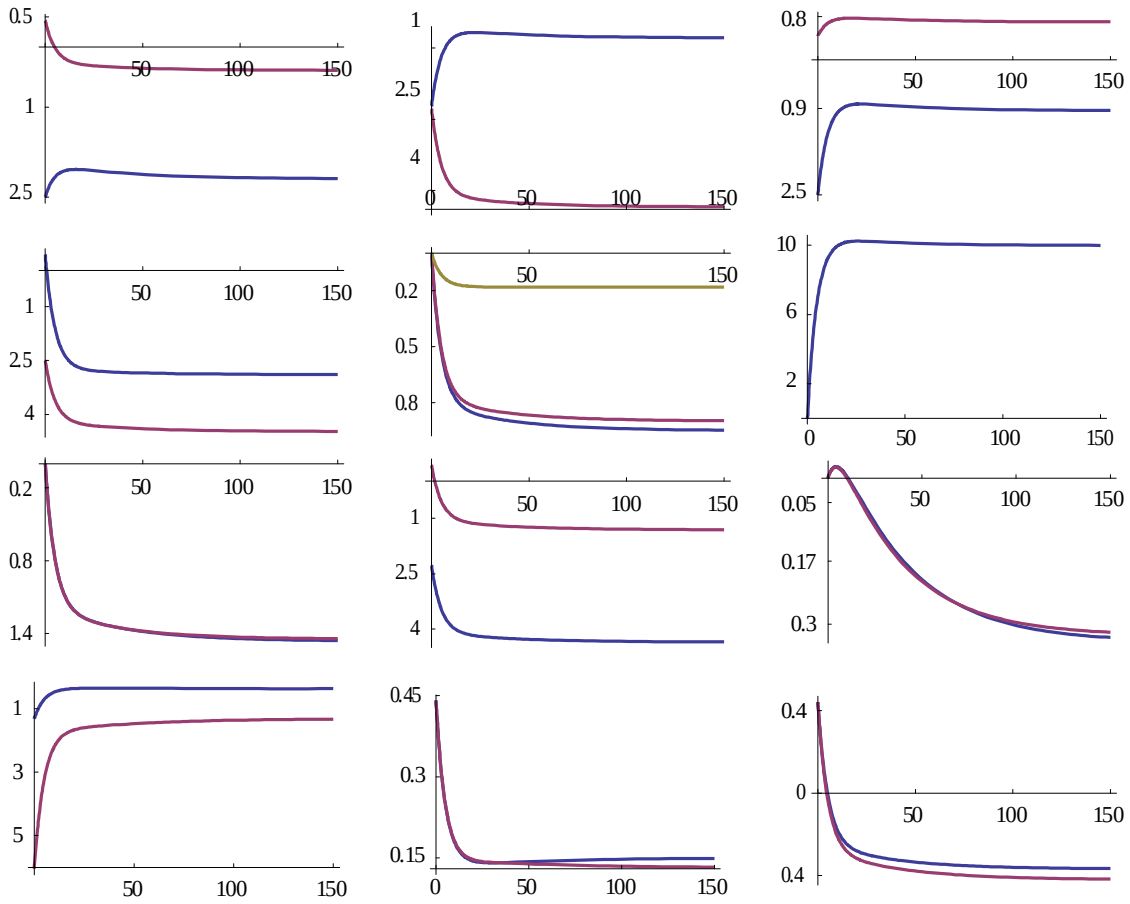

Figure 7. The Propensity to Save Being Reduced

5. Concluding Remarks

This paper extended the well-known Uzawa-Lucas two-sector growth model to include gender division of labor, leisure and education with gender discrimination in society. The production and economic structures, human capital accumulation are based on the Uzawa-Lucas model, while the utility function and gender division of labor, leisure time and study time are based on the model by Zhang (2014). This study was mainly concerned with impact of gender discrimination. We emphasized the impact of the gender discrimination, gender-differentiated preferences and human capital utilization efficiencies upon the gender-differentiated time distribution, economic growth, economic structure, and human capital and wage rates. We took account of learning by

education in modeling human capital accumulation and gender discrimination in the education sector. As the model is analytically too complicated to get explicit solutions, we simulated the model to demonstrate existence of equilibrium points and motion of the national economy. We also conducted comparative dynamic analysis in regard to some parameters. The model can be extended in different directions. For instance, like Funke and Strulik (2000) who propose a formal framework to integrate the two separate lines of research on growth with knowledge – the Uzawa model with education and the endogenous growth models, we may also take account of research in modelling economic growth with gender differences. It should be also mentioned that Lacopetta (2010) examines the transitional economic dynamics with education and innovation. In order to properly describe behavior of the household, we need to take account of these differences. Studies show that there are gender differences in social preferences (Croson and Gneezy, 2009; Eckel and Grossman, 1998, 2001), risk preferences (Charness and Gnezy, 2012; Dwyer et al., 2002; Jianakoplos and Bernasek, 1998; Dittrich and Leipold, 2014; and bargaining behavior (Dittrich et al., 2014). It is important to investigate what insights we may get by introducing some of these gender differences into our analytical framework.

Appendix: Proving the Lemma

We now show that the dynamics can be expressed by three dimensional differential equations. From (3) and (6), we obtain

$$z \equiv \frac{r+\delta_k}{w_1} = \frac{h_i N_i}{K_i} = \frac{h_e N_e}{K_e}, \tag{A1}$$

where

$$h_i(H_1) \equiv \frac{\alpha_i}{\beta_i H_1^{\theta_1}}, \quad h_e(H_1) \equiv \frac{\alpha_e}{\beta_e H_1^{\theta_1}}.$$

From (3), we have

$$w_2 = \frac{w_1 H_2^{\theta_2}}{(1-\phi) H_1^{\theta_1}}. \tag{A2}$$

From (2) and (3), we solve

$$r = \alpha_i A_i \left(\frac{z}{h_i}\right)^{\beta_i} - \delta_k, \quad w_1 = \beta_i A_i H_1^{\theta_1} \left(\frac{z}{h_i}\right)^{-\alpha_i}. \tag{A3}$$

From (4) and (5), we have

$$\hat{p} = \left(\frac{r + \delta_k}{\alpha_e A_e}\right)\left(\frac{K_e}{N_e}\right)^{\beta_e}. \tag{A4}$$

Insert (A1) in (A4)

$$\hat{p}(z, H_1, H_2) = \left(\frac{r + \delta_k}{\alpha_e A_e}\right)\left(\frac{h_e}{z}\right)^{\beta_e}. \tag{A5}$$

From (14) we have

$$\hat{p} = \left(1 + \frac{\tilde{T}_2 \bar{N}}{\tilde{T}} \phi_e\right) p. \tag{A6}$$

From (8) and the definition of $\tilde{T}$ we have

$$\frac{\tilde{T}_2 \bar{N}}{\tilde{T}} = \frac{\eta_2 / \bar{p}_2}{\eta_1 / p_1 + \eta_2 / \bar{p}_2}. \tag{A7}$$

Insert the definitions of $\bar{p}_2$ in (A7)

$$\frac{\tilde{T}_2 \bar{N}}{\tilde{T}} = \frac{\eta_2}{(\bar{\phi}_e p + \bar{\phi} w_2)\eta_1 / (p + w_1) + \eta_2}. \tag{A8}$$

Substituting (A8) into (A6) yields

$$p^2 + f_1 p - f = 0, \tag{A9}$$

where

$$f_1(z, H_1, H_2) \equiv \frac{\eta_1 \bar{\phi} w_2 + \eta_2 w_1 + w_1 \eta_2 \phi_e - \bar{\phi}_e \hat{p} \eta_1 - \eta_2 \hat{p}}{\phi_e \eta_1 + \eta_2 + \eta_2 \phi_e},$$

$$f(z, H_1, H_2) \equiv \frac{\phi w_2 \hat{p} \eta_1 + w_1 \eta_2 \hat{p}}{\phi_e \eta_1 + \eta_2 + \eta_2 \phi_e} > 0.$$

Solve (A9) with p as variable

$$p(z, H_1, H_2) = -\frac{f_1}{2} + \sqrt{\frac{f_1^2}{4} + f}. \tag{A10}$$

Here we neglect the negative solution of (A9) as it is economically not meaningful. We see that we can consider r, w_1, w_2, $\hat{p}$, and p as functions of z, H_1 and H_2.

From (8), we have

$$\bar{T}_1 + \bar{T}_2 = W\bar{y}, \tag{A11}$$

$$\tilde{T}_1 + \tilde{T}_2 = P\bar{y}. \tag{A12}$$

where

$$\bar{y} = (r+1)\bar{k} + W_0, \quad W_0 \equiv w_1 T_0 + \phi w_2 T_0, \quad W \equiv \frac{\sigma_1}{w_1} + \frac{\sigma_2}{w_2}, \quad P \equiv \frac{\eta_1}{\bar{p}_1} + \frac{\eta_2}{\bar{p}_2}.$$

From $T_q + \tilde{T}_q + \bar{T}_q = T_0$, we have

$$T_1 + T_2 + \tilde{T}_1 + \tilde{T}_2 + \bar{T}_1 + \bar{T}_2 = 2T_0.$$

Insert (A11) and (A12) in the above equation

$$T_1 + T_2 = W_1 - W_2\bar{k}, \tag{A13}$$

where

$$W_1 \equiv 2T_0 - (P+W)W_0, \quad W_2 \equiv (P+W)(r+1).$$

From (11) and (3) we have

$$\tilde{T}_1 + \tilde{T}_2 = \frac{A_e K_e}{N}\left(\frac{z}{h_e}\right)^{\beta_e}. \tag{A14}$$

where we also use (A1). Insert (A12) into (A14)

$$K_e = R\bar{k} + \tilde{W}, \tag{A15}$$

where we use

$$\tilde{W}(z, H_1, H_2) \equiv \frac{W_0 P \bar{N}}{A_e}\left(\frac{h_e}{z}\right)^{\beta_e}, \quad R(z, H_1, H_2) \equiv \frac{(r+1)P\bar{N}}{A_e}\left(\frac{h_e}{z}\right)^{\beta_e}.$$

From (A15), $K = \bar{k}\bar{N}$ and $K_i + K_e = K$

$$K_i = \left[1 - \frac{(r+1)P}{A_e}\left(\frac{h_e}{z}\right)^{\beta_e}\right]\bar{N}\bar{k} - W_0\frac{P\bar{N}}{A_e}\left(\frac{h_e}{z}\right)^{\beta_e}. \tag{A16}$$

By (A1) we have

$$N_i = \frac{zK_i}{h_i}, \quad N_e = \frac{zK_e}{h_e}. \tag{A17}$$

From (A17) and $K_i + K_e = \bar{k}\bar{N}$

$$N = N_i + N_e = \frac{\bar{k}z\bar{N}}{h_i} + hK_e, \tag{A18}$$

where

$$h(z, H_1, H_2) \equiv \left(\frac{1}{h_e} - \frac{1}{h_i}\right)z.$$

From the definition of N and (A18) we have

$$\frac{\bar{k}z}{h_i} + \frac{h}{N}K_e = H_1^{\theta_1}T_1 + H_2^{\theta_2}T_2.$$

Insert (A15) in the above equation

$$\left(\frac{z}{h_i} + \frac{hR}{\bar{N}}\right)\bar{k} + \frac{h\tilde{W}}{\bar{N}} = H_1^{\theta_1}T_1 + H_2^{\theta_2}T_2. \tag{A19}$$

Solve (A13) and (A19) with work hours as variables

$$\begin{aligned} T_1 &= W_1 - W_2\bar{k} - T_2, \\ T_2 &= \left(\frac{z}{h_i} + \frac{hR}{\bar{N}} + H_1^{\theta_1}W_2\right)\frac{\bar{k}}{H_2^{\theta_2} - H_1^{\theta_1}} + \frac{h\tilde{W}/N - H_1^{\theta_1}W_1}{H_2^{\theta_2} - H_1^{\theta_1}}. \end{aligned} \tag{A20}$$

From (12) and (2) we get

$$c + s - \delta \bar{k} = \frac{A_i K_i^{\alpha_i} N_i^{\beta_i}}{N}, \tag{A21}$$

where $\delta \equiv 1 - \delta_k$. Insert (A1) and (8) in (A21)

$$\left[(\xi + \lambda)(r + 1) - \delta\right]\bar{k} + (\xi + \lambda)W_0 = \frac{m K_L}{N}, \tag{A22}$$

where we also use $\bar{y} = (r + 1)\bar{k} + W_0$ and

$$m(z, H_1, H_2) \equiv A_i \left(\frac{z}{h_i}\right)^{\beta_i}.$$

Substituting (A16) into (A22) yields

$$\bar{k} = \bar{\Lambda}(z, H_1, H_2)$$
$$\equiv W_0 \left[\xi + \lambda + \frac{mP}{A_e}\left(\frac{h_e}{z}\right)^{\beta}\right]\left[m + \delta - \frac{(r-1)mP}{A_e}\left(\frac{h_e}{z}\right)^{\beta} - (\xi + \lambda)(r + 1)\right]^{-1}. \tag{A23}$$

We showed r, w_1, w_2, $\hat{p}$, and p as functions of z, H_1 and H_2. By (A23), we express $\bar{k}$ as functions of z, H_1 and H_2. By (A20) we express T_1 and T_2 as functions of z, H_1 and H_2. By its definition we express N as a function of z, H_1 and H_2. By the definition of $\bar{y}$ and $K = kN$, we express K and $\bar{y}$ as functions of z, H_1 and H_2. By (A15) and (A16) we solve K_e and K_i. By (A1) we solve N_e and N_i. We solve F_i and F_e as functions of z, H_1 and H_2. From (8), we solve c, s, $\tilde{T}_q$ and $\bar{T}_q$ as functions of z, H_1 and H_2. From (15) it is straightforward to show that the motion of human capital can be expressed as functions of z, H_1 and H_2 at any point in time

$$\dot{H}_q = \Omega_q(z, H_1, H_2), \quad q = 1, 2. \tag{A24}$$

We now show that changes in $z(t)$ can be expressed as a differential equation with z, H_1 and H_2 as variables. First, from (10) it is straightforward to express the change of wealth as a function of z, H_1 and H_2 as follows

$$\dot{\bar{k}} = \Omega_0(z, H_1, H_2) \equiv s - \bar{k}. \tag{A25}$$

Taking derivatives of $\bar{k} = \bar{\Lambda}$ with respect to time, we have

$$\dot{\bar{k}} = \frac{\partial \bar{\Lambda}}{\partial z}\dot{z} + \Omega_1 \frac{\partial \bar{\Lambda}}{\partial H_1} + \Omega_2 \frac{\partial \bar{\Lambda}}{\partial H_2}, \tag{A26}$$

where we use (A24). Substituting (A25) into (A26) yields

$$\dot{z} = \Omega(z, H_1, H_2) \equiv \left(\Omega_0 - \Omega_1 \frac{\partial \bar{\Lambda}}{\partial H_1} - \Omega_2 \frac{\partial \bar{\Lambda}}{\partial H_2}\right)\left(\frac{\partial \bar{\Lambda}}{\partial z}\right)^{-1}. \tag{A27}$$

The three differential equations, (A27) and (A22), contain three variables z, H_1 and H_2. We thus proved the lemma.

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